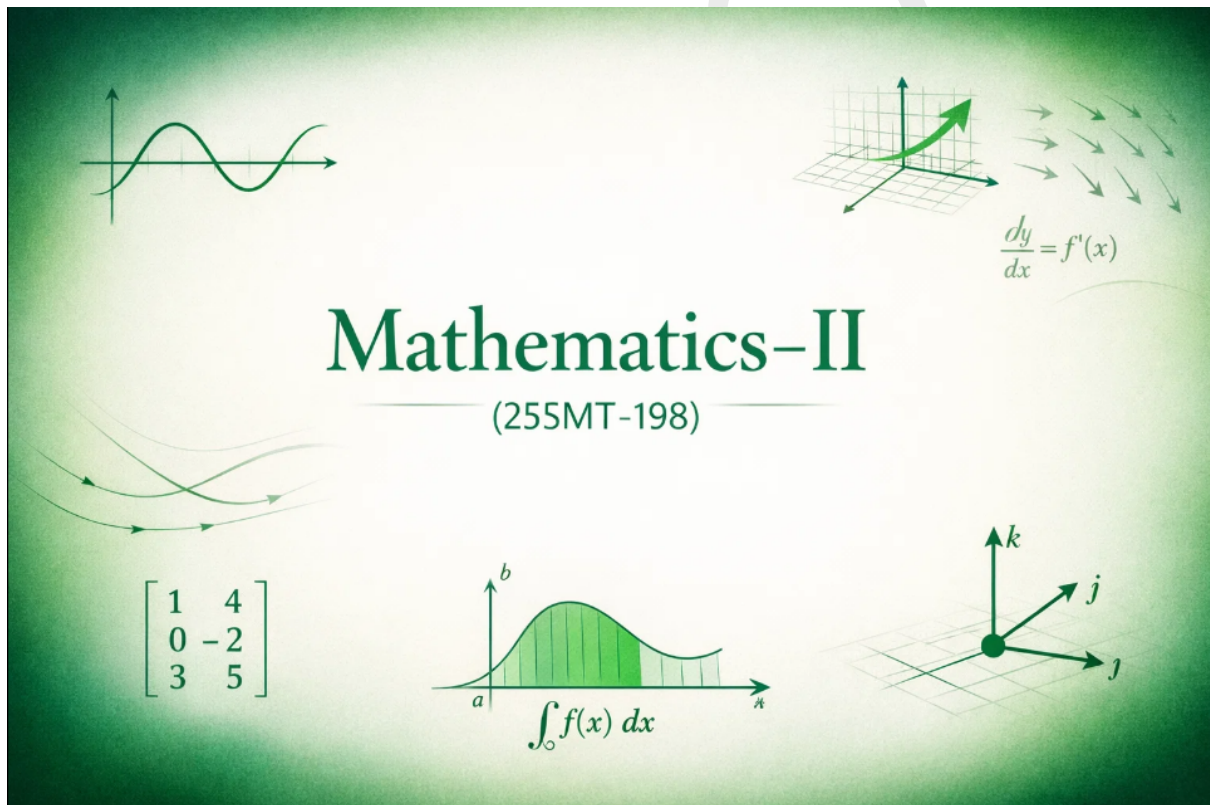


Mathematics-II (25SMT-198)

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Syllabus for Unit 1

Mathematics-II (25SMT-198)

- **Set Theory**

- Introduction to set theory
- Types and classes of sets
- Finite and infinite sets
- Set operations (union, intersection, difference, complement)
- Algebra of sets
- Principle of duality
- Venn diagrams and their applications
- Power sets
- Minsets and Maxsets
- Cartesian product of sets
- Principle of inclusion and exclusion

- **Relations and Functions**

- Introduction to relations
- Binary relations
- Types of relations
- Equivalence relations and partitions
- Partial order relations
- Introduction to functions
- Types of functions
- Composition of functions
- Inverse functions
- Mathematical induction

- **Propositional Calculus**

- Introduction to propositions
- Logical connectives: AND ($\wedge$), OR ($\vee$), NOT ($\neg$)



- Compound statements
- Truth values of compound propositions
- Truth tables
- Tautologies
- Contradictions

• **Self-Study Topics**

- Cardinal arithmetic
- Advanced concepts of functions

Notefity

Chapter 1

Set Theory

Set Theory is a fundamental branch of mathematics that deals with the collection of objects. It forms the basis of various areas in computer science, including data structures, databases, logic, probability, and artificial intelligence. This chapter introduces the basic concepts of set theory, types of sets, operations on sets, and the algebra governing set operations.

1.1 Introduction to Set Theory

A **set** is a well-defined collection of distinct objects, called elements.

Definition: A set is a collection of objects where it is clearly determined whether an object belongs to the collection or not.

Representation of Sets:

- **Roster Form:** Listing all elements Example: $A = \{1, 2, 3, 4\}$
- **Set Builder Form:** Describing properties of elements.
Example: $A = \{x \mid x \text{ is a natural number less than } 5\}$

Common Symbols:

- $\in$: belongs to
- $\notin$: does not belong to
- $\subseteq$: subset

Doubt Clarification: Order of elements does not matter in a set, and repetition is not allowed.



1.2 Types and Classes of Sets

Sets can be classified based on their elements and properties.

Important Types of Sets:

- **Empty (Null) Set:** A set with no elements Example: $\emptyset$
- **Singleton Set:** A set with exactly one element Example: $\{5\}$
- **Equal Sets:** Sets having exactly the same elements Example: $\{1, 2, 3\} = \{3, 2, 1\}$
- **Subset:** A set whose elements are contained in another set Example: $A \subseteq B$
- **Universal Set:** The set containing all possible elements under consideration

Important: Every set is a subset of itself and the empty set is a subset of every set.

1.3 Finite and Infinite Sets

Finite Set: A set containing a limited or countable number of elements.

Example: $A = \{1, 2, 3, 4\}$

Infinite Set: A set containing an unlimited number of elements.

Example: Set of natural numbers: $\mathbb{N} = \{1, 2, 3, \dots\}$

Key Difference: Finite sets can be completely listed, infinite sets cannot.

1.4 Set Operations

Set operations are used to combine or relate sets.

Union

The **union** of two sets contains all elements that belong to either set.

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

Example: $A = \{1, 2, 3\}, B = \{3, 4, 5\}$ $A \cup B = \{1, 2, 3, 4, 5\}$



Intersection

The **intersection** contains elements common to both sets.

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

Example: $A \cap B = \{3\}$

Difference

The **difference** of two sets contains elements present in the first set but not in the second.

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

Example: $A - B = \{1, 2\}$

Complement

The **complement** of a set contains elements not present in the set but present in the universal set.

$$A' = U - A$$

Note: Complement is always defined with respect to a universal set.

1.5 Algebra of Sets

The algebra of sets defines laws that govern set operations, similar to algebraic laws.

Important Laws:

- **Commutative Laws:** $A \cup B = B \cup A$, $A \cap B = B \cap A$
- **Associative Laws:** $(A \cup B) \cup C = A \cup (B \cup C)$
- **Distributive Laws:** $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- **Identity Laws:** $A \cup \emptyset = A$, $A \cap U = A$
- **De Morgan's Laws:** $(A \cup B)' = A' \cap B'$, $(A \cap B)' = A' \cup B'$

Exam Tip: De Morgan's laws are frequently tested in problem-solving questions.



1.6 Principle of Duality

The **principle of duality** is an important concept in set theory which states that every valid algebraic identity remains valid when certain operations and elements are interchanged.

Principle of Duality: If an identity involving sets is true, then its dual statement is also true.

Duality Transformations:

- Replace union ($\cup$) with intersection ($\cap$)
- Replace intersection ($\cap$) with union ($\cup$)
- Replace universal set (U) with empty set ($\emptyset$)
- Replace empty set ($\emptyset$) with universal set (U)

Example: Given identity:

$$A \cup \emptyset = A$$

Its dual is:

$$A \cap U = A$$

Important: Complements remain unchanged while applying the principle of duality.

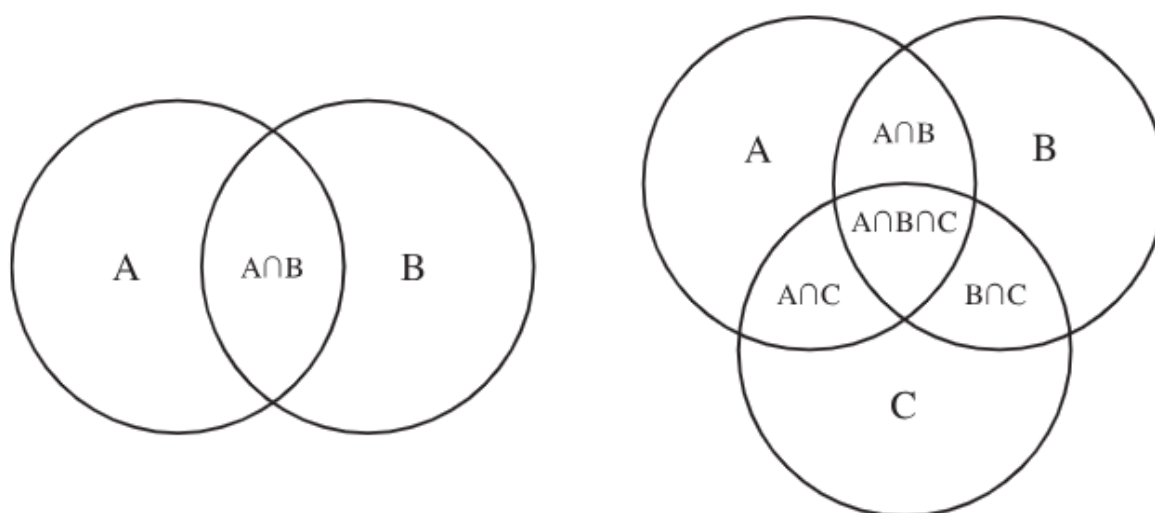
Significance:

- Reduces the number of identities to be proved
- Helps in verifying set-theoretic laws easily

1.7 Venn Diagrams and Their Applications

A **Venn diagram** is a pictorial representation of sets using closed curves, usually circles.

Definition: A Venn diagram visually represents the relationships between different sets.



Basic Elements of Venn Diagrams:

- Rectangle representing the universal set
- Circles representing individual sets
- Overlapping regions showing common elements

Applications of Venn Diagrams:

- Visualizing set operations such as union and intersection
- Solving problems involving multiple conditions
- Understanding relationships among datasets

Illustrative Example: Let A be the set of students who study Mathematics and B be the set of students who study Physics. $A \cap B$ represents students who study both subjects.

Exam Tip: Venn diagrams are commonly used in numerical and word problems.

1.8 Power Sets

The **power set** of a given set is the collection of all possible subsets of that set.

Definition: If A is a set, then its power set $P(A)$ is the set of all subsets of A .

Example: If $A = \{a, b\}$, then:

$$P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$$

**Number of Elements in a Power Set:**

$$|P(A)| = 2^n$$

where n is the number of elements in set A .

Solved Example: If a set contains 3 elements, then:

$$|P(A)| = 2^3 = 8$$

Key Insight: Every element of a power set is itself a set.

1.9 Minsets and Maxsets

In set theory, **minsets** and **maxsets** are concepts used to describe the smallest and largest sets with respect to inclusion under given conditions.

Minset (Minimal Set): A set is called a minset if it does not properly contain any other set satisfying the given condition.

Maxset (Maximal Set): A set is called a maxset if it is not properly contained in any other set satisfying the given condition.

Key Difference:

- A **minset** cannot be reduced further
- A **maxset** cannot be expanded further

Illustrative Example: Let $U = \{1, 2, 3, 4\}$ and consider all subsets containing element 1.

- Minset: $\{1\}$
- Maxset: $\{1, 2, 3, 4\}$

Doubt Clarification: A minimal set is not necessarily unique, and a maximal set is not necessarily the universal set.

1.10 Cartesian Product of Sets

The **Cartesian product** of two sets is the set of all ordered pairs formed by taking one element from each set.



Definition: If A and B are two sets, then their Cartesian product is

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

Example: Let $A = \{1, 2\}$ and $B = \{x, y\}$

$$A \times B = \{(1, x), (1, y), (2, x), (2, y)\}$$

Important Observations:

- Order of elements matters in ordered pairs
- $A \times B \neq B \times A$ in general

Cardinality:

$$|A \times B| = |A| \times |B|$$

Exam Tip: Cartesian products form the basis for defining relations and functions.

1.11 Principle of Inclusion and Exclusion

The **Principle of Inclusion and Exclusion (PIE)** is used to compute the number of elements in the union of overlapping sets.

Principle: To find the size of the union of sets, add the sizes of individual sets and subtract the sizes of their intersections.

For Two Sets:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

For Three Sets:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$



Solved Example: In a class:

- 30 students study Mathematics
- 25 study Physics
- 10 study both

Number of students studying at least one subject:

$$|M \cup P| = 30 + 25 - 10 = 45$$

Key Insight: The principle avoids double-counting elements that belong to multiple sets.

Chapter 2

Relations and Functions

Relations and functions extend the concept of sets by describing how elements of one set are associated with elements of another set. They play a crucial role in mathematics, logic, computer science, and database systems. This chapter introduces relations, binary relations, their types, and the important concept of equivalence relations and partitions.

2.1 Introduction to Relations

A **relation** defines a connection between elements of two sets.

Definition: A relation from a set A to a set B is a subset of the Cartesian product $A \times B$.

Notation: If $(a, b) \in R$, then element a is said to be related to element b under relation R .

Example: Let $A = \{1, 2, 3\}$ and $B = \{2, 4, 6\}$. A relation $R = \{(1, 2), (2, 4), (3, 6)\}$ relates each element of A to its double in B .

Doubt Clarification: A relation does not need to include all elements of a set.



2.2 Binary Relations

A **binary relation** is the most common type of relation, involving pairs of elements.

Binary Relation: A binary relation R on a set A is a subset of $A \times A$.

Example: Let $A = \{1, 2, 3\}$. Relation $R = \{(1, 1), (2, 2), (3, 3)\}$ is a binary relation on A .

Representation of Binary Relations:

- Set of ordered pairs
- Matrix representation
- Directed graph (digraph)

Key Insight: Binary relations are used extensively in databases, graphs, and state transition systems.

2.3 Types of Relations

Binary relations can be classified based on their properties.

Reflexive Relation: A relation R on set A is reflexive if $(a, a) \in R$ for all $a \in A$.

Symmetric Relation: A relation R is symmetric if $(a, b) \in R$ implies $(b, a) \in R$.

Antisymmetric Relation: A relation R is antisymmetric if $(a, b) \in R$ and $(b, a) \in R$ implies $a = b$.

Transitive Relation: A relation R is transitive if $(a, b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$.

Example: The relation " $\leq$ " on integers is reflexive, antisymmetric, and transitive.

Exam Tip: Clearly state the definition and verify properties using ordered pairs.

2.4 Equivalence Relations and Partitions

An **equivalence relation** is a special type of relation that groups elements into equivalence classes.



Equivalence Relation: A relation that is reflexive, symmetric, and transitive.

Example: Let R be the relation “congruence modulo 3” on integers. Two integers a and b are related if $a \equiv b \pmod{3}$.

Equivalence Class: The equivalence class of an element a is:

$$[a] = \{x \mid (x, a) \in R\}$$

Partition of a Set:

- A partition divides a set into disjoint, non-empty subsets
- Every element belongs to exactly one subset

Key Relationship: Every equivalence relation defines a partition, and every partition defines an equivalence relation.

2.5 Partial Order Relations

A **partial order relation** is a binary relation that defines an ordering among elements, though not all elements must be comparable.

Partial Order Relation: A relation R on a set A is a partial order if it is *reflexive*, *antisymmetric*, and *transitive*.

Properties Recalled:

- Reflexive: $(a, a) \in R$
- Antisymmetric: $(a, b), (b, a) \in R \Rightarrow a = b$
- Transitive: $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$

Numerical Example 1:

Let $A = \{1, 2, 3\}$ and $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (1, 3), (2, 3)\}$.

Verification:

- Reflexive: All (a, a) are present
- Antisymmetric: No (b, a) exists for $a \neq b$
- Transitive: $(1, 2)$ and $(2, 3)$ imply $(1, 3)$



Hence, R is a **partial order relation**.

Poset: (A, R) is called a *partially ordered set*.

Exam Insight: Most numerical questions ask you to verify the three properties step by step.

2.6 Introduction to Functions

A function is a restricted form of relation where each input has exactly one output.

Function: A function $f : A \rightarrow B$ is a relation such that every element of A is mapped to exactly one element of B .

Key Terminology:

- Domain: Set of inputs
- Codomain: Set of possible outputs
- Range: Actual outputs produced

Illustrative Example:

Let $A = \{1, 2, 3\}$, $B = \{2, 4, 6\}$ and $f = \{(1, 2), (2, 4), (3, 6)\}$.

Each element of A has exactly one image in B . Hence, f is a valid function.

Doubt Clarification: If even one element of the domain maps to two outputs, the relation is *not* a function.

2.7 Types of Functions

One-to-One (Injective) Function

A function is injective if distinct elements of the domain map to distinct elements of the codomain.

Example: $f : \{1, 2, 3\} \rightarrow \{4, 5, 6\}$ $f = \{(1, 4), (2, 5), (3, 6)\}$

No two inputs share the same output. Hence, injective.

Many-to-One Function

Two or more domain elements map to the same codomain element.

Example: $f(x) = x^2$ from $\mathbb{Z} \rightarrow \mathbb{Z}$ $f(2) = 4$ and $f(-2) = 4$

Hence, many-to-one.



Onto (Surjective) Function

A function is onto if every element of the codomain is mapped by at least one domain element.

Example: $f : \{1, 2, 3\} \rightarrow \{2, 4, 6\}$ $f = \{(1, 2), (2, 4), (3, 6)\}$

All elements of codomain are covered. Hence, onto.

Into Function

A function whose range is a proper subset of the codomain.

Example: $f : \{1, 2\} \rightarrow \{2, 4, 6\}$ $f = \{(1, 2), (2, 4)\}$

Element 6 has no pre-image. Hence, into.

Bijjective Function

A function that is both one-to-one and onto.

Example: $f(x) = x + 1$ from $\mathbb{Z} \rightarrow \mathbb{Z}$

Each input has a unique output and every integer is covered.

Key Result: Only bijective functions have inverse functions.

Worked Numerical

Question: Let $f : \{1, 2, 3\} \rightarrow \{a, b, c\}$ be defined as $f = \{(1, a), (2, b), (3, b)\}$. Determine the type of function.

Solution:

- Two elements map to $b \Rightarrow$ many-to-one
- Element c has no pre-image $\Rightarrow$ not onto

Conclusion: f is a **many-to-one into function**.

2.8 Composition of Functions

The composition of functions is a way of combining two functions to form a new function.

Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be two functions. The **composition of f and g** , denoted by $g \circ f$, is defined as:

$$(g \circ f)(x) = g(f(x))$$

The function $g \circ f$ maps elements from set A directly to set C .



Condition for Composition: The range of the first function must be a subset of the domain of the second function.

Example: Let

$$f(x) = 2x, \quad g(x) = x + 3$$

Then,

$$(g \circ f)(x) = g(2x) = 2x + 3$$

and

$$(f \circ g)(x) = f(x + 3) = 2x + 6$$

Hence,

$$g \circ f \neq f \circ g$$

Important Properties:

- Composition of functions is associative
- Composition of functions is not commutative

2.9 Inverse Functions

An inverse function reverses the mapping of a given function.

If $f : A \rightarrow B$ is a bijective function, then its inverse function $f^{-1} : B \rightarrow A$ exists such that:

$$f^{-1}(f(x)) = x \quad \text{for all } x \in A$$

Example: Let

$$f(x) = 3x + 2$$

To find the inverse:

$$y = 3x + 2$$

$$x = \frac{y - 2}{3}$$

Thus,

$$f^{-1}(x) = \frac{x - 2}{3}$$

Key Observations:

- Only one-to-one and onto (bijective) functions have inverses
- The inverse function undoes the effect of the original function

Verification:

$$f(f^{-1}(x)) = x \quad \text{and} \quad f^{-1}(f(x)) = x$$



2.10 Mathematical Induction

Mathematical induction is a powerful technique used to prove statements that are true for all natural numbers.

Principle of Mathematical Induction:

To prove that a statement $P(n)$ is true for all natural numbers $n \geq 1$, the following two steps are required:

Step 1: Base Case Verify that $P(1)$ is true.

Step 2: Induction Step Assume that $P(k)$ is true for some arbitrary natural number k . Then prove that $P(k + 1)$ is also true.

If both steps are satisfied, then $P(n)$ is true for all $n \in \mathbb{N}$.

Example: Prove that:

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

Base Case ($n = 1$):

$$1 = \frac{1(1+1)}{2} = 1$$

Induction Hypothesis: Assume the formula holds for $n = k$:

$$1 + 2 + \cdots + k = \frac{k(k+1)}{2}$$

Induction Step:

$$\begin{aligned} 1 + 2 + \cdots + k + (k+1) &= \frac{k(k+1)}{2} + (k+1) \\ &= \frac{(k+1)(k+2)}{2} \end{aligned}$$

Hence, the statement is true for $n = k + 1$.

Therefore, by mathematical induction, the formula holds for all natural numbers n .

Chapter 3

Propositional Calculus

Propositional Calculus, also known as propositional logic, is a branch of logic that deals with propositions and their combinations using logical connectives. It provides the formal foundation for reasoning, mathematical proofs, digital circuits, and artificial intelligence.

3.1 Introduction to Propositions

A **proposition** is a declarative statement that is either true or false, but not both.

Examples of Propositions:

- “The sun rises in the east.” (True)
- “5 is a prime number.” (True)
- “7 is an even number.” (False)

Non-Examples:

- “Close the door.” (Command)
- “What time is it?” (Question)

Each proposition is usually represented by a propositional variable such as p , q , or r .

Truth Value: The truth value of a proposition is either **True (T)** or **False (F)**.

3.2 Logical Connectives

Logical connectives are symbols used to combine one or more propositions to form compound statements.



Connective Symbols	Words	Terms	Example
$\wedge$	AND	Conjunction	$A \wedge B$
$\vee$	OR	Disjunction	$A \vee B$
$\rightarrow$	Implies	Implication	$A \rightarrow B$
$\Leftrightarrow$	If and only If	Biconditional	$A \Leftrightarrow B$
$\neg$	Not	Negation	$\neg A$

Logical AND ($\wedge$)

The conjunction of two propositions p and q , written as $p \wedge q$, is true if and only if both p and q are true.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Logical OR ($\vee$)

The disjunction of two propositions p and q , written as $p \vee q$, is true if at least one of p or q is true.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Logical NOT ($\neg$)

The negation of a proposition p , written as $\neg p$, reverses its truth value.

p	$\neg p$
T	F
F	T



Example: If p is “It is raining”, then $\neg p$ is “It is not raining”.

3.3 Compound Statements

A **compound statement** is formed by combining two or more propositions using logical connectives.

Example: Let p : “It is raining” q : “The ground is wet”

Then:

- $p \wedge q$: “It is raining and the ground is wet”
- $p \vee q$: “It is raining or the ground is wet”
- $\neg p$: “It is not raining”

Truth Value of Compound Statements: The truth value of a compound statement depends on the truth values of its component propositions and the connectives used.

Example: If $p = T$ and $q = F$, then:

$$p \wedge q = F, \quad p \vee q = T$$

3.4 Truth Values of Compound Propositions

The truth value of a compound proposition depends on:

- The truth values of its component propositions
- The logical connectives used to combine them

Each logical connective follows fixed rules that determine how truth values are computed.

Example: Let p : “5 is greater than 3” (True) q : “5 is an even number” (False)

Then:

$$p \wedge q = F, \quad p \vee q = T, \quad \neg p = F$$

Thus, even though q is false, the compound proposition $p \vee q$ is true because at least one component is true.

3.5 Truth Tables

A **truth table** is a systematic way to list all possible truth values of propositions and evaluate the truth values of compound statements.

Steps to Construct a Truth Table:



- List all possible combinations of truth values for the propositions
- Apply logical connectives to compute compound expressions

Truth Table for $p \wedge q$:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth Table for $p \vee q$:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

3.6 Tautologies

A **tautology** is a compound proposition that is true for all possible truth values of its components.

Example:

$$p \vee \neg p$$

Truth Table:

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

Since the final column contains only true values, the proposition is a tautology.

3.7 Contradictions

A **contradiction** is a compound proposition that is false for all possible truth values of its components.

Example:

$$p \wedge \neg p$$

Truth Table:



p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

Since the final column contains only false values, the proposition is a contradiction.

Notefity



Self-Study Topics

Cardinal Arithmetic

Cardinal arithmetic deals with the arithmetic operations performed on the cardinalities (sizes) of sets. It extends the concept of counting to finite and infinite sets.

For finite sets, cardinal arithmetic follows the usual rules of arithmetic. If two finite sets A and B are disjoint, then:

$$|A \cup B| = |A| + |B|$$

and

$$|A \times B| = |A| \cdot |B|$$

For infinite sets, cardinal arithmetic behaves differently. For example, the set of natural numbers $\mathbb{N}$ and the set of even numbers have the same cardinality, even though the latter is a proper subset of the former. This illustrates that infinite sets can be put into one-to-one correspondence with their proper subsets.

Cardinal arithmetic is important for understanding concepts such as countability, uncountability, and the size of infinite data structures.

Advanced Concepts of Functions

Advanced concepts of functions extend basic function theory and are widely used in mathematics and computer science.

Important topics include:

- **Composite Functions:** Combining two or more functions into a single function.
- **Inverse Functions:** Functions that reverse the mapping of another function.
- **Recursive Functions:** Functions defined in terms of themselves.
- **Partial Functions:** Functions that are not defined for every element of the domain.



Mid Semester Test 1 – Sample Paper

Total Marks : 20

Time Allotted : 1 hour

Instructions:

- The question paper consists of 2 sections: Section A and Section B.
- Section A has 5 questions of 2 marks each and Section B has 2 questions of 5 marks each.

Section A ($5 \times 2 = 10$ marks)

1. If A and B are finite sets such that $|A| = 5$, $|B| = 3$, and $|A \cap B| = 2$, find $|A \cup B|$.
2. Using suitable set identities, prove that:

$$(A \cap B)' = A' \cup B'$$

3. Explain why the empty set is considered a subset of every set.
4. Define a partial order relation. With a suitable example, explain why every partial order is not a total order.
5. Construct the truth table for the compound proposition $(p \wedge q) \vee \neg p$ and state whether it is a tautology, contradiction, or contingency.

Section B ($2 \times 5 = 10$ marks)

6. Explain the principle of inclusion and exclusion. In a class of 60 students, 35 study Mathematics, 30 study Physics, and 15 study both. Find the number of students who study at least one of the two subjects.
7. Define relations and explain equivalence relations with a suitable example. Using mathematical induction, prove that:

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$



Answer Key

Section A ($5 \times 2 = 10$ marks)

1. If A and B are finite sets such that $|A| = 5$, $|B| = 3$, and $|A \cap B| = 2$, find $|A \cup B|$.

Answer:

Using the principle of inclusion and exclusion:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Substituting the given values:

$$|A \cup B| = 5 + 3 - 2 = 6$$

Answer: $|A \cup B| = 6$

2. Using suitable set identities, prove that $(A \cap B)' = A' \cup B'$.

Answer:

Let $x \in (A \cap B)'$.

Then,

$$x \notin A \cap B$$

which implies,

$$x \notin A \text{ or } x \notin B$$

Hence,

$$x \in A' \text{ or } x \in B'$$

Therefore,

$$x \in A' \cup B'$$

Thus,

$$(A \cap B)' \subseteq A' \cup B'$$

Similarly, let $x \in A' \cup B'$.

Then,

$$x \in A' \text{ or } x \in B'$$



which implies,

$$x \notin A \cap B$$

Hence,

$$x \in (A \cap B)'$$

Therefore,

$$(A \cap B)' = A' \cup B'$$

3. Explain why the empty set is considered a subset of every set.

Answer:

A set A is a subset of set B if every element of A is also an element of B .

Since the empty set $\emptyset$ has no elements, there is no element that violates this condition.

Hence, the empty set is a subset of every set.

4. Define a partial order relation. With a suitable example, explain why every partial order is not a total order.

Answer:

A relation R on a set A is called a **partial order relation** if it is:

- Reflexive
- Antisymmetric
- Transitive

Example: The relation " $\subseteq$ " on the power set of a given set is a partial order.

In a partial order, not all elements are comparable. A total order requires that every pair of elements be comparable.

Hence, every partial order is not a total order.



5. Construct the truth table for $(p \wedge q) \vee \neg p$ and state whether it is a tautology, contradiction, or contingency.

Answer:

p	q	$p \wedge q$	$\neg p$	$(p \wedge q) \vee \neg p$
T	T	T	F	T
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T

Since the final column contains both true and false values, the given proposition is a **contingency**.

Section B ($2 \times 5 = 10$ marks)

6. Explain the principle of inclusion and exclusion. In a class of 60 students, 35 study Mathematics, 30 study Physics, and 15 study both. Find the number of students who study at least one of the two subjects.

Answer:

The principle of inclusion and exclusion is used to find the number of elements in the union of overlapping sets.

For two sets A and B :

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Given:

$$|M| = 35, \quad |P| = 30, \quad |M \cap P| = 15$$

$$|M \cup P| = 35 + 30 - 15 = 50$$

Answer: 50 students study at least one of the two subjects.



7. Define relations and explain equivalence relations with a suitable example. Using mathematical induction, prove that:

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Answer:

A **relation** from set A to set B is a subset of the Cartesian product $A \times B$.

An **equivalence relation** is a relation that is:

- Reflexive
- Symmetric
- Transitive

Example: The relation “congruence modulo n ” on integers is an equivalence relation.

Proof by Mathematical Induction:

Base Case ($n = 1$):

$$1^2 = \frac{1(1+1)(2 \cdot 1 + 1)}{6} = 1$$

Induction Hypothesis: Assume the statement is true for $n = k$:

$$1^2 + 2^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6}$$

Induction Step:

$$\begin{aligned} & 1^2 + 2^2 + \dots + k^2 + (k+1)^2 \\ &= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \\ &= \frac{(k+1)(2k^2 + 7k + 6)}{6} \\ &= \frac{(k+1)(k+2)(2k+3)}{6} \end{aligned}$$

Thus, the statement holds for $n = k + 1$.

Therefore, by mathematical induction,

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

is true for all natural numbers n .

Closing Note

I sincerely hope that these notes and the preparatory questions included alongside them prove helpful in strengthening your understanding and boosting your confidence for the examination you are preparing for.

As you revise, try not to rush through the content—pause at examples, work out the numericals on your own, and revisit sections that feel slightly uncomfortable. That discomfort usually signals learning in progress. Use these notes as a guide and a companion, not as something to be memorized line by line.

Wishing you the very best for your exams—may your preparation feel clear, structured, and calm when it matters most.

For future updates, refinements, and additional resources, keep checking **Noteify**:

<https://subhayu18.github.io/Noteify/>

If you have any doubts, suggestions, or simply wish to connect, feel free to reach out to me on LinkedIn:

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You can also use the direct contact options provided on the Noteify website itself.

Regards,
Subhayu